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Seminário

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Non occurrence of the Lavrentiev phenomenon for scalar multi-dimensional variational problems

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Let $f : \mathbb{R}^n \to \mathbb{R}$ be a convex function, Ω be an open and bounded subset of $\mathbb{R}^n$. We consider the functional

$$I(u) := \int_\Omega f(\nabla u(x))\, dx \quad u \in W^{1,1}(\Omega).$$

It is known [2] that if Ω is star-shaped then the Lavrentiev phenomenon does not occur if one does not consider a fixed boundary datum, i. e.

$$\inf\{I(u) : u \in W^{1,1}(\Omega)\} = \inf\{I(u) : u \in W^{1,\infty}(\Omega)\}.$$

The importance of the non occurrence of the Lavrentiev phenomenon is due to the fact that only in that case, the methods of numeric analysis allow to approximate the infimum value of the operator (finite elements method). When the boundary datum is taken into account, in spite of the paradigm saying that the Lavrentiev phenomenon should not occur, there are just a few results corroborating the statement, apart the obvious case where some "natural growth conditions" are assumed: in a recent paper Cellina and Bonfanti [1] proved that if the lagrangian is radial and both the boundary datum and he domain are of class $\mathcal{C}^2$ then the Lavrentiev phenomenon does not occur.

In a work in progress, jointly with Pierre Bousquet and Giulia Treu, we make a considerable step forward in favor of the above conjecture and take into account a wider class of domains and lagrangians, with a minimum set of assumptions (in particular no growth conditions!): its description is the main argument of the lecture.

References

[1] G. Bonfanti and A. Cellina, *On the non-occurrence of the lavrentiev phenomenon*, Adv. Calc.Var. **6** (2013), 93–121.

[2] G. Buttazzo and M. Belloni, *A survey on old and recent results about the gap phenomenon in the calculus of variations*, Recent developments in well-posed variational problems, 1995, pp. 1–27.