Centro de Investigação em Matemática e Aplicações
Departamento de Matemática

Seminário

30 de Novembro de 2011, Quarta-feira
CLAV – 138 - 16:00 horas

Boundary layer problem: Navier-Stokes equations and Euler equations

Nikolai V. Chemetov
CMAF/Universidade de Lisboa

Resumo

We consider the Navier-Stokes equations in a bounded domain $\Omega \subseteq \mathbf{R}^2$

$$\begin{aligned} \mathbf{v}_t + \mathrm{div}\,(\mathbf{v} \otimes \mathbf{v}) - \nabla p &= \mu \triangle \mathbf{v}, \qquad \mathbf{x} \in \Omega, \quad t > 0, \\ \mathrm{div}\,\mathbf{v} &= 0, \\ \mathbf{v}(\mathbf{x}, 0) &= \mathbf{v}_0(\mathbf{x}), \qquad \mathbf{x} \in \Omega, \end{aligned}$$

admitting flows through the boundary $\partial\Omega$ of Ω

$$\begin{aligned} \mathbf{v} \cdot \mathsf{n} &= a(\mathbf{x}, t), \qquad \mathbf{x} \in \partial\Omega, \quad t > 0, \\ 2D(\mathbf{v})\mathsf{n} \cdot \mathsf{s} + \alpha(\mathbf{x}, t)\, \mathbf{v} \cdot \mathsf{s} &= b(\mathbf{x}, t). \end{aligned}$$

The last one are so-called Navier slip boundary conditions. Here $\mathbf{v}(\mathbf{x}, t)$ - the velocity of the fluid; $p(\mathbf{x}, t)$ - the pressure; $D(\mathbf{v}) := \frac{1}{2}[\nabla \mathbf{v} + (\nabla \mathbf{v})^T]$ - the rate-of-strain tensor of $\mathbf{v}$; (n, s) - the pair formed by the outside normal and tangent vectors to the boundary Γ of Ω.

The main result: *When $\mu \to 0$ we shown that the solutions $\mathbf{v}_\mu$ of the Navier - Stokes equations converge to the solution $\mathbf{v}$ of the Euler equations, satisfying the Navier slip boundary conditions on the part of the boundary $\partial\Omega$, where $\mathbf{v} \cdot \mathsf{n} = a < 0$, such that*

$$\mathbf{v}_\mu \to \mathbf{v} \qquad \textit{strongly in} \qquad L_\infty(0, T; W_p^1(\Omega)). \tag{1}$$

This result solved a so-called problem of boundary layers.

References

Chemetov N.V., Cipriano F., "Boundary layer problem: Navier-Stokes equations and Euler equations", submitted.