Centro de Investigação em Matemática e Aplicações
Departamento de Matemática

Seminário

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Algebraic structures in Digital Geometry

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Resumo

Since the 70's Imaging algorithmics has led to the emergence of new disciplines such as Digital Topology and Digital Geometry. Despite the many works and results about discrete objects (subsets of $\mathbb{Z}^n$), it remains difficult to organize them according to the schemes of Analysis or Differential Geometry. However, they naturally connect to Arithmetics and Algebra.

We will discuss three kinds of relations between discrete objects and algebra.

1. Digital lines and planes and the special linear group $Sl(n,\mathbb{Z})$.

This is the study of relationships between objects defined by inequalities

$$\gamma \leq p.x < \gamma + \varepsilon,$$

where $p, x \in \mathbb{Z}^n$, $\gamma \in \mathbb{Z}$, $\varepsilon \in \mathbb{N}$ and $p.x$ denotes scalar product, with the special linear group $Sl(n,\mathbb{Z})$, particularly in the cases and $n = 2$ and $n = 3$.

2. The Algebra of Ehrhart quasi-polynomials.

If P denotes the ring of periodic sequences $\mathbb{Z} \to \mathbb{C}$, the algebra of polynomials $\mathfrak{P}[x]$ formalizes the notion of quasi-polynomial introduced by E. Ehrhart about forty years ago for integer points counting. We shall see that these quasi-polynomials are closely related to the discretization of polynomials with rational coefficients.

3. Multidimensional continued fractions (also called Arnold-Klein sails).

By stating in the early 90's, that Lagrange's theorem about the periodicity of continued fractions of quadratic irrationals could be generalized to higher dimensions by combining (the almost forgotten) F. Klein's theory of multidimensional continued fractions with Dirichlet's units theorem, V.I. Arnold has renewed interest in the study of this subject. We shall explain the first convincing example (due to E. Korkina and G. Lachaud) and the current work about the sails of hyperbolic operators with cyclic Galois group.